

Bornholdt's spin model of a market dynamics in high dimensions

Takuya Yamano*

Institute for Theoretical Physics, Cologne University, D-50923 Köln, Euroland

E-mail: ty@thp.Uni-Koeln.DE

Received (received date)

Revised (revised date)

We present results of an extension of the market model introduced by Bornholdt to high dimensions. Three and four dimensions are shown to behave similar to two, for suitable parameters.

Keywords: market dynamics; Bornholdt model; return distribution; volatility; autocorrelation function.

1. Introduction

To construct a reasonable dynamical financial market model consistent with the observed properties in real market is a central theme. Many models have been proposed along this line. Specifically application of concepts and techniques of statistical mechanics are giving deeper insight into the understanding of the complex behavior of a market. Fluctuations of prices of commodities, stocks and foreign exchange rates are examples of these. Reproduction of the distribution of returns based on the empirical data are a strong criterion to select models. It is important to set a communication structure for the model of financial market. Various models are proposed to imitate the real market communication. Among them we mention the minority game¹, percolation^{2,3}, Ising model^{4,5,6,7,8,9} and so forth. In this paper we focus on a spin model that seems to be appropriate to capture the movements of constituent agents in a simplified manner. In previous work, Iori⁵ has modeled three possible states of each agent's decision as spin-1 model and its decision making is affected by two different kind of noises and their nearest neighbors.⁵ However in this model the influence of market prices on each trader decision is not incorporated. Bornholdt recently formulated these influences with two different scales of local interaction and global one in two dimension.⁹ A metastable phase appears

*On leave from Department of Applied Physics, Faculty of Science, Tokyo Institute of Technology, Oh-okayama, Meguro-ku Tokyo, 152-8551, Japan
E-mail: tyamano@mikan.ap.titech.ac.jp

at randomly frozen finite magnetization which correspond to a bubble-like state in terms of economics.⁹

We assume that the traders live on a d-dimensional lattice. This totally contrasts the case⁶ where the *super-spins* do not need any geometry. In this paper we present the results of higher dimensions in the Bornholdt model. This can be motivated from the fact that one of the most important characters of the market evolution is the heterogeneous structure of agents. The heterogeneity arises from not only the geometrical distance of agents but also the amount of knowledge that each agents has, accessibility of market information prevailing, preferences, and processing skills for investments etc.

2. The Bornholdt Model

In the model, the spin variables are either $+1$ or -1 , which allows each agent to decide actions of buying or selling at each time step t . Thus the magnetization $M(t) = \sum_{j=1}^N S_j(t)/N$ can be interpreted as a measure of price.⁹ We identify the logarithm of the absolute value of the magnetization as *returns*, i.e. $ret(t) = \ln |M(t)| - \ln |M(t-1)|$. The use of linear returns (or simply relative price change) is also possible and in fact the main properties are also seen for this definition.¹⁰ Each spin is updated by the following heat bath dynamics,

$$S_i(t+1) = \begin{cases} +1 & \text{with } p = [1 + \exp(-2\beta h_i(t))]^{-1} \\ -1 & \text{with } 1-p \end{cases}, \quad (1)$$

where β is the inverse temperature and $h_i(t)$ is a time-dependent local field that each spin feels. This corresponds to a *signal* that each agent i receives at time t ⁵ and also to "individual bias" of the i -th agent.⁶ The *temperature* is introduced in a totally parallel way to the Ising spin model in magnetism. This fictitious temperature has been used in the previous works.^{6,7}

Bornholdt introduced the following form of the local field,

$$h_i(t) = \sum_{j=i}^N J_{ij} S_j(t) - \alpha S_i(t) \left| \frac{1}{N} \sum_{j=1}^N S_j(t) \right| \quad (2)$$

The above form can be considered incorporating two contradictory movements of constituent agents in a real market: herd behavior and preference of minority. J_{ij} denotes the strength of the communication between the agents i and j . Furthermore J_{ij} is set to a constant J for nearest neighbor agents and zero for all the other pairs. This term corresponds to the "disagreement function" of the i -th agent.⁶ α is a strength of the coupling to the magnetization and assumed to be positive. In a spin language, the first term prefers the ferromagnetic state. The second term, on the other hand, tends to encourage a spin flip when magnetizations becomes large.⁹

The more complicated form of the local field is given as⁹

$$h_i(t) = \sum_{j=i}^N J_{ij} S_j(t) - \alpha C_i(t) \frac{1}{N} \sum_{j=1}^N S_j(t), \quad (3)$$

meaning we have two spins on each site at each time: demand spin $S_i(t)$ and strategy spin $C_i(t)$ of an agent i . Consideration in a market context gives an interpretation of strategy spin as⁹

$$C_i(t) = \begin{cases} +1 & \text{anti-ferro (fundamentalist)} \\ -1 & \text{ferro (chartist)} \end{cases} \quad (4)$$

This reminds us of the Lux-Marchesi model¹¹, where agents are divided into these two groups and evolve with interacting mutually and moreover with changing their strategies. We flip the strategy spin at next time step: $C_i(t+1) = -C_i(t)$ if the $S_i(t)$ is antiparallel to the global coupling to the magnetization i.e.,

$$S_i(t) C_i(t) \frac{1}{N} \sum_{j=1}^N S_j(t) < 0 \quad (5)$$

is satisfied at each time because a positive energy contribution (risky or preference being minority) to the dynamics encourages these fundamentalists to switch their strategies to chartists and vice versa. These updates are done after updating $S_i(t)$.

3. Simulations and Results

We performed the simulation with the reduced version of the local field eq.(2) except for dynamical evolutions of the fundamentalists-chartists behavior. The same behavior is observed either with eq.(3) or with eq.(2). The size of the hyper cubic lattice was set at $L = 101, 21$, and 7 for the dimension $d = 2, 3$ and 4 respectively. We have chosen the temperature (in units of J/k_B) $T = 1.5, 4.0$ and 6.0 for the dimension $d = 2, 3$ and 4 respectively. Note that these temperature is below the critical temperature $T_c = 2.269(2d), 4.511(3d)$ and $6.680(4d)$ in the case of $\alpha = 0$. This selection of parameters allows us to observe the intermittent behavior in temporal evolution of $ret(t)$ or clustered volatilities. Fig.1 shows the distribution of returns for three different dimensions, whose shapes are non-Gaussian. The volatility of the returns or the generalized cumulative absolute returns¹⁴ can be measured with the following quantity^{13,14}

$$V(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} |ret(t')|^\gamma, \quad (6)$$

where n is the number of data sampled and γ is a real exponent. We set $n = 1$ and $\gamma = 1$, which leads to the cumulative distribution of absolute returns (Fig.2). The measured exponents of these curves are $-1.15, -1.41$ and -1.50 respectively

using the intermediate region of $8 \dots 150$, $15 \dots 150$ and $40 \dots 150$ for fitting, where power-law scaling can be seen. A volatility clustering can be seen as a slow decay of the autocorrelation function of absolute returns in Fig.3 on a log-log plot. Fig.4. is the replot of the same data obtained in our simulation. The slope of the each curves is determined as -1.85×10^{-3} , -1.80×10^{-2} and -7.90×10^{-2} with the fitting time $350 \dots 1000(2d)$, $40 \dots 140(3d)$ and $5 \dots 35(4d)$ respectively, where the accurate value changes as the fitting regions vary. Fig.4 shows the ratio of the difference between the number of fundamentalists and that of chartists to the total number of agents with eq(3) in the 4d case. The decrease in the ratio of fundamentalists corresponds to high variance in changes of returns.

4. Discussion and Summary

In this paper, we have performed simulations of the recently proposed model by Bornholdt in hyper cubic lattices. We confirmed that there is no intermittent behavior in return(magnetization) when we employ only the second term of eq.(2). This means that market intermittency emerge as a consequence of a frustration across two different scales: local connection with neighbours and global coupling with market.⁹ The effects of dimensionality was confirmed, which exhibits similar behavior for three different cases. Moreover we confirmed that the similar intermittent behavior in returns appear when we drop out S_i in the second term of the local field (eq.(2)). In this case, however, the fictitious temperature of the market becomes higher than that of the original one($T = 7.5, 9.3$ and 11.0 in the $2d$, $3d$, and $4d$ case respectively with the same other parameter values as in the text). However their temperatures exceed the critical ones. The exponent of cumulative distribution of returns is one of the ongoing discussion points for many models.^{3,15} The recent measurement of the exponent based on a empirical data (S&P 500)¹² says the asymptotic slope of close to -3 . A percolation model provides -2.9 with 201×201 lattice³ and -4 in high dimensions. In this sense, percolation model gives reasonable distribution. On the one hand, the model of Biham *et al.* provided the range of $-2.5 \dots -3.5$ for the tails of the distribution,¹⁵ which depends on the number of agents and a factor characterizing the market dynamics. The slope (moreover the shape) of the cumulative distribution is strongly affected by the temperature in the present model. We confirmed the effect in the 3d case by fixing the other parameters. This means the power-law scaling in intermediate regions is very sensitive to the temperature. More specifically, cumulative distributions have a shoulder when the temperature is not the *tuned* value. The intermittency becomes more sparse as the temperature descends. The same can be seen as the lattice size becomes large. As for autocorrelation function, the empirical data are well fitted with the exponent -0.3 according to Gopikrishnan et al.¹² and Liu et al.¹³. The exponent determined from minority game¹ is -0.64 . In the present model, the tails of the autocorrelation function do not behave like power laws but as exponential up to four dimensions.

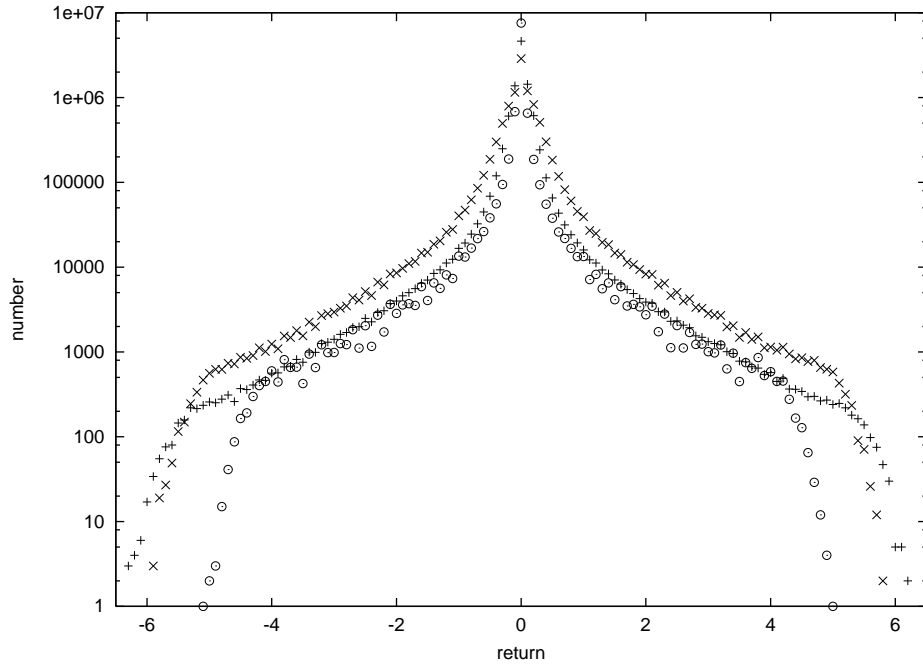


Fig. 1. Distribution of the return defined as the difference of the logarithm of the absolute magnetization. Parameters $J = 1.0$ and $\alpha = 4.0$ are common for all dimensions. The symbols \odot , + and \times correspond to $(d, T) = (2, 1.5)$, $(3, 4.0)$, and $(4, 6.0)$ respectively.

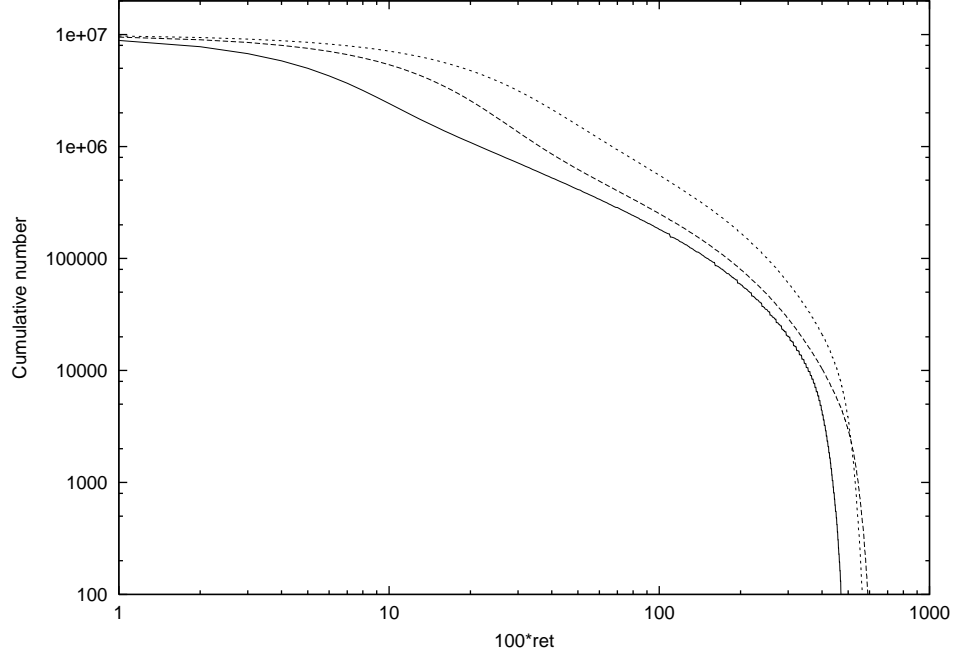


Fig. 2. Cumulative distribution of absolute returns multiplied by 100. For all lines, $J = 1.0, \alpha = 4.0$. The solid, broken and dotted line correspond to $2d$ with $T = 1.5$, $3d$ with $T = 3.8$ and $4d$ with $T = 6.0$ respectively. For better statistics, we have summed over 10^7 Monte Carlo steps.

Acknowledgements

We acknowledge the financial support from the DAAD (Deutscher Akademischer Austauschdienst) for staying the Universität zu Köln, where this work was suggested by D. Stauffer. We also thank S. Bornholdt for his kind explanation of his paper.

1. D. Challet, A. Chessa, M. Marsili and Y-C. Zhang, *Quantitative Finance*, **1**, 168 (2001) and references there in.
2. R. Cont and J.P. Bouchaud, *Macroeconomic Dynamics* **4**, 170 (2000).
3. For recent review: D. Stauffer, *Adv. Complex. Syst.* **4**, 19 (2001) and references there in.
4. J. Cremer, *Physica A* **246**, 377 (1997).
5. G. Iori, *Int. J. Mod. Phys. C* **10**, 1149 (1999).
6. D. Chowdhury and D. Stauffer, *Eur. Phys. J. B* **8**, 477 (1999).
7. B.M. Roehner and D. Sornette, *Eur. Phys.J.B* **16**, 729 (2000).

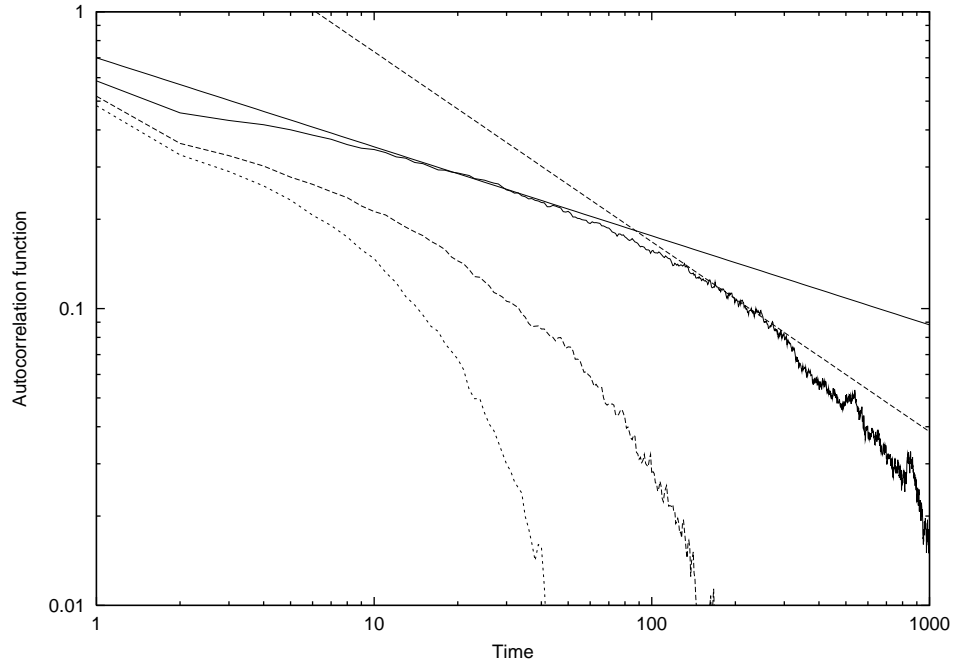


Fig. 3. Autocorrelation function of absolute returns calculated from 10^6 steps. The curves correspond to the $4d$, $3d$ and $2d$ from left to right. The solid straight line denotes the slope -0.3 from references ^{12,13}. The broken straight line is a slope -0.64 from a model of minority game¹.

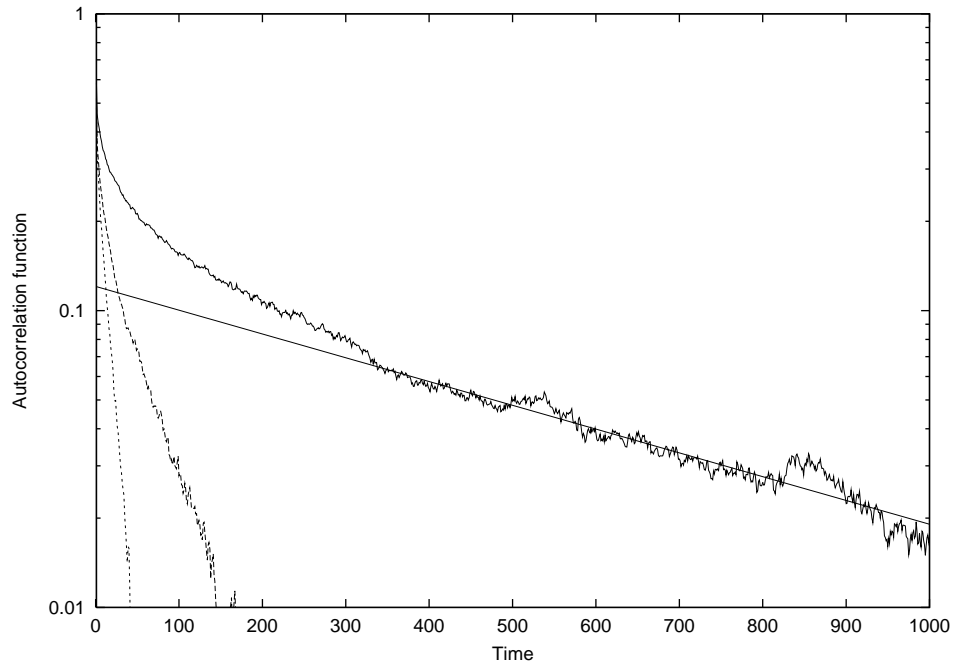


Fig. 4. Autocorrelation function of absolute returns in semi-log scale replotted. The fitted straight line to the $2d$ data has exponent -1.85×10^{-3} .

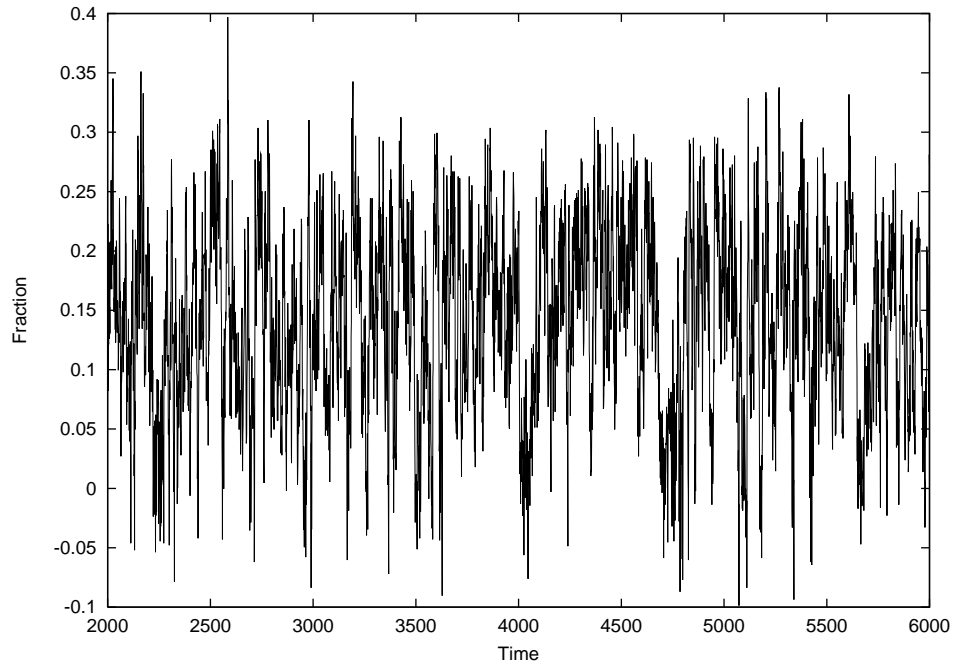


Fig. 5. The change of the market constituents in $4d$ case ($J = 1.0$, $\alpha = 4.0$ and $T = 6.0$) at a period of 4000.

T. Yamano

8. T. Kaizoji, cond-mat/0010263.
9. S. Bornholdt, Int. J. Mod. Phys.C **12**, 667 (2001).
10. Private communication with S. Bornholdt.
11. T. Lux and M. Marchesi, Nature **397**, 498 (1999).
12. P. Gopikrishnan, M. Meyer, L.A.N. Amaral and H.E. Stanley, Eur. Phys.J.B **3**, 139 (1998); P. Gopikrishnan, V. Plerou, L.A.N. Amaral, M. Meyer and H.E. Stanley, Phys.Rev.E, **60**, 5305 (1999).
13. Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C-K. Peng and H.E. Stanley, Phys.Rev.E, **60**, 1390 (1999).
14. M. Pasquini and M. Serva, cond-mat/9810232; cond-mat/9903334; R. Baviera, M. Pasquini, M. Serva, D. Vergni and A. Vulpiani, cond-mat/9901225.
15. O. Biham, Z-F. Huang, O. Malcai and S. Solomon, Phys. Rev. E, **64** 026102 (2001).